Stability and Robustness of Dynamical Traffic Networks

Modeling Task Force Meeting
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Outline of the Talk

- Motivation
- From static to dynamical traffic networks
- Dynamics = capacity constraints + route choice + traffic control
- Stability and resilience
- Conclusion and future work

Motivation

- Costs of traffic congestion [TTI TAMU urban mobility report 2012]
  - Financial cost: $ 121 Billion
  - Time wastage: 5.5 Billion hours
  - Health, environment, etc.

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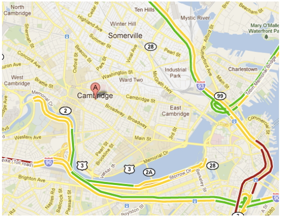

Typical monday at 18:30

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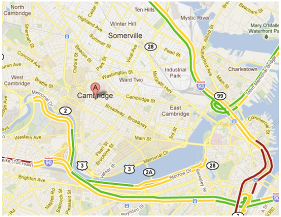

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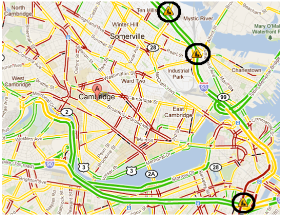
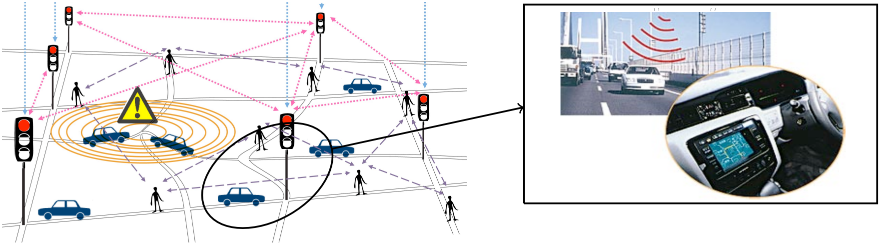
disruption →



Monday 11/07/11 at 18:30

From centralized to distributed traffic control



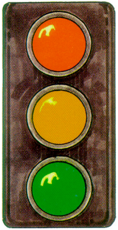
Why distributed ?

- Increased resilience to failure of control modules
- Scalability with respect to network size
- On-board computation
- Trade-off between performance and distributedness

Key elements of traffic models

infrastructure capacity

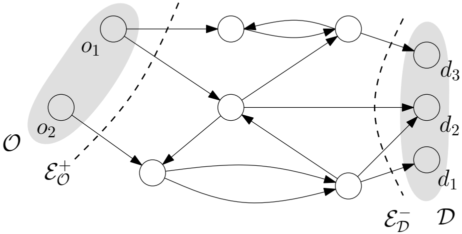
driver choice



traffic light

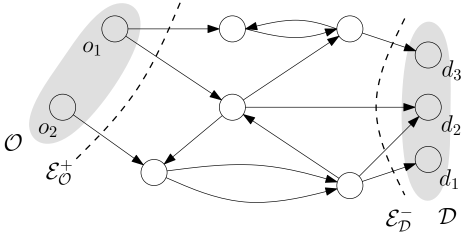
congestion pricing

Network flow



- Flow capacity on every link
- Flow conservation at every node
- Maximum feasible load = bottle-neck capacity

Network flow



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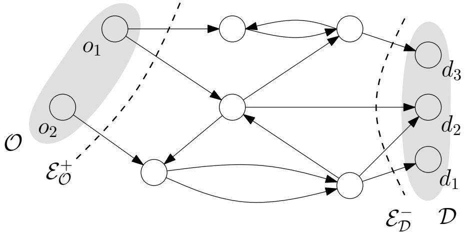
- Static framework
- Centralized

Network flow



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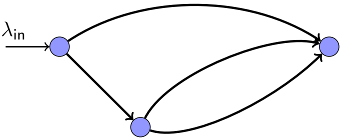
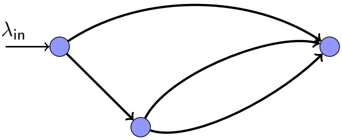
Framework of choice for planning purposes

Congestion games



- Traffic distribution is the outcome of a non-cooperative game between drivers
- Driver decisions are dynamic
- Driver decisions are myopic

Congestion games



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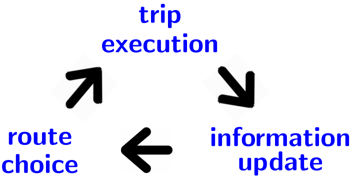


- Equilibrium outcome
- Adaptability to disturbances

- Static
- Global decision dynamics

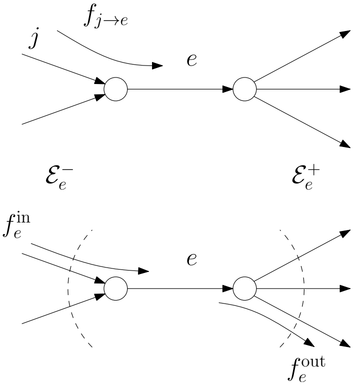
From static to dynamical model

- $f_{j \to e}$ flow routed from j to e
-

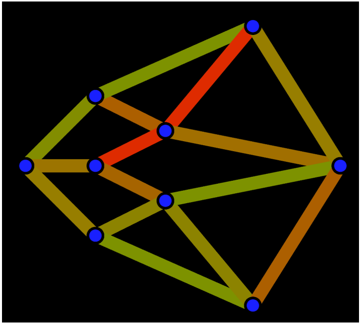
$$f_e^{\text{out}} = \sum_{\text{outgoing } j} f_{e \to j}$$

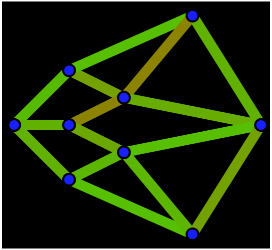
-

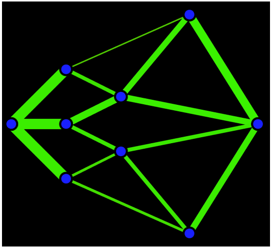
$$f_e^{\text{in}} = \sum_{\text{incoming } j} f_{j \to e}$$



Stability and resilience of transportation networks



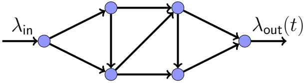




Quantifying stability and resilience

Stability

- Network is stable if output equals input
- For unstable networks, delay is infinite
- Response to 'small' disturbances



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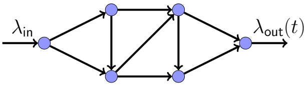


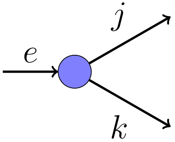
Resilience

- Link disturbance = loss in capacity
- Network disturbance = $\sum$ link disturbances
- Smallest malicious disturbance that destabilizes the network

Influence of route choice decisions

$$f_{e \to j} = D_e \, G_j$$

G_j: fraction of drivers choosing link j



Cooperative route choice decisions

- business as usual congestion $\implies$ business as usual decision

$$G_j^*(\text{eqm}) = \text{eqm route choice}$$

- choose links with less congestion

$$\frac{\partial G_j^*}{\partial \rho_k} \geq 0$$

- Example: i-logit
- utility$_i$ = myopia + inertia

Cooperative route choice decisions

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- If the load on the system is feasible, then G^* is stabilizing
- Within the constraint of not controlling the inflow, G^* performs best
- G^* does not give the maximum possible resilience
- The gap increases with the network size

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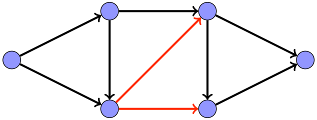
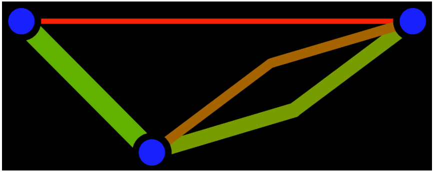
Resilience = min node residual capacity



Examples of suboptimal route choice

- passive routing



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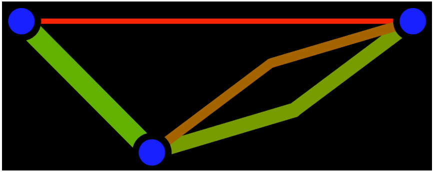
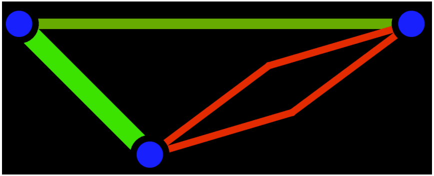
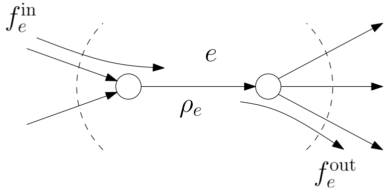


- aggressive routing



Back to the general case

$$\dot{\rho}_e = f_e^{\text{in}} - f_e^{\text{out}}$$



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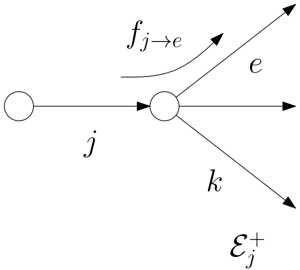
Cooperative routing

Boundary conditions

- Empty link $\implies$ no outflow
- No flow towards congested links
- Fully congested links give maximum outflow if there is room downstream



Cooperative routing

- increase in congestion $\implies$ increase in outflow

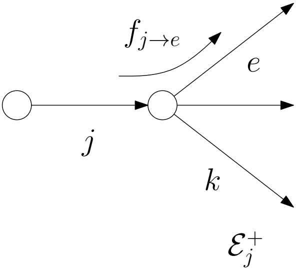
  $$\frac{\partial f_{j\to e}}{\partial \rho_j} \geq 0$$

- avoid congested links

  $$\frac{\partial f_{j\to e}}{\partial \rho_k} \geq 0$$

- increase in downstream congestion $\implies$ decrease in outflow

  $$\frac{\partial f_j^{\text{out}}}{\partial \rho_k} \leq 0$$



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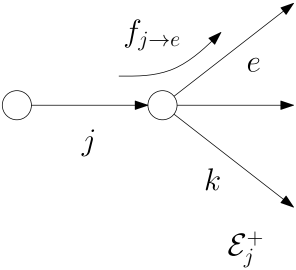
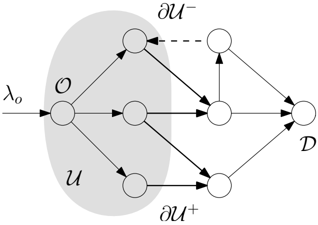


- Control based on local information
- Backward propagation of information

Performance of cooperative routing

- Feasible load $\implies$ network is stable
- Infeasible load $\implies$ there exists a unique bottleneck which gets jammed simultaneously.
- Entire network is shut down or no link is jammed



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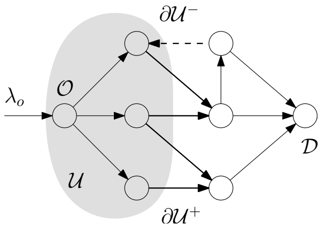


- Maximum possible network stability and resilience
  Resilience = network residual capacity
- Graceful failure

Implications for planning

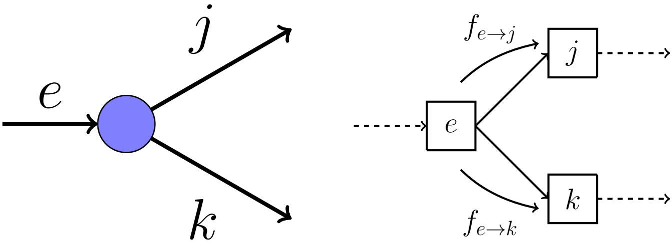
- Quantitative framework for resilience
- Dependence of resilience on traffic load, network structure, link capacity and route choice behavior
- Resilience as a social objective for transportation planning
- Resilience not aligned with typical social objectives such as delay

Current and future work

- Comprehensive study of resilience under a variety of practical constraints on traffic flow
- From analysis to control of traffic flow
- Connection between agent-based and macroscopic models
- Tradeoff between resilience and delay
- Extension to other infrastructure networks

Traffic flow theory

Cell Transmission Model for Networks:

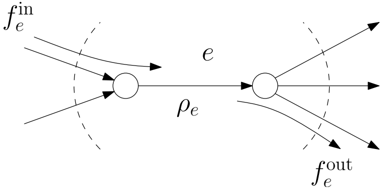


- Outflow from link e depends on congestion on j and k
- Ratio between $f_{e\to j}$ and $f_{e\to k}$ is independent of congestion on j and k

From static to dynamical model

Mass conservation

$$\dot{\rho}_e = f_e^{\text{in}} - f_e^{\text{out}}$$



Constraints

- Density capacity on every link
- Flow capacity on every link

- f_e^{in} and f_e^{out} depend on $\underbrace{\text{traffic flow, route choice and signal control}}_{\textbf{dynamic}}$